

Lecture 5 Summary

Tues 05/08/14

Vocabulary

- * topological space; topology *(not work)*
- * open set
- * closed set
- * open ball
- * neighbourhood
- * neighbourhood filter
- * interior
- * closure
- * interior point
- * close point/adherent point

terminology

topology

Examples

TOPOLOGIES

(1) Let X be a set.

The discrete topology on X is:

$$\tau = \{\text{all subsets of } X\}$$

(2) Let (X, d) be a metric space.

The metric space topology on X is:

$$\tau = \{U \subseteq X \mid U \text{ is a union of open balls}\}$$

(3) Let (X, τ) be a topological space. Let $Y \subseteq X$.

The subspace topology on Y is:

$$\tau_Y = \{U \cap Y \mid U \in \tau\}$$

(4) Let (X, τ_X) , (Y, τ_Y) be topological spaces.

The product topology on $X \times Y$ is:

$$\tau_{X \times Y} = \{U \subseteq X \times Y \mid U \text{ is the union of } A \times B, \text{ with } A \in \tau_X, B \in \tau_Y\}$$

OPEN & CLOSED SETS

Let $X = \mathbb{R}$ with the metric given by:

$$d(x, y) = |x - y|$$

and the metric space topology.

(1) $(a, b) = \{x \in \mathbb{R} \mid a < x < b\} = B_{\frac{b-a}{2}}(\frac{a+b}{2})$ is open.

(2) $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$ is closed.

(3) $\mathbb{A}[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$ is not open, not closed.

(4) \emptyset and \mathbb{R} are both open and closed.

Homework

- Show that examples (1)-(4) really define topologies

1) $\{ \emptyset, \mathbb{R} \}$ ~~not topo~~
 $\{ \emptyset, \mathbb{R}, \{x\} \}$
 $\{ \emptyset, \mathbb{R}, \{x\}, \{y\} \}$
 $\{ \emptyset, \mathbb{R}, \{x\}, \{y\}, \{x, y\} \}$
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2) solutionA31241090T $\{ \emptyset, \mathbb{R} \times \{x\}, \{x\} \times \mathbb{R} \}$

$\{ \emptyset, \mathbb{R} \times \{x\}, \{x\} \times \mathbb{R}, \{x, y\} \times \{x, y\} \}$

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